MICRO-428: Metrology

Week Two: Elements of Statistics

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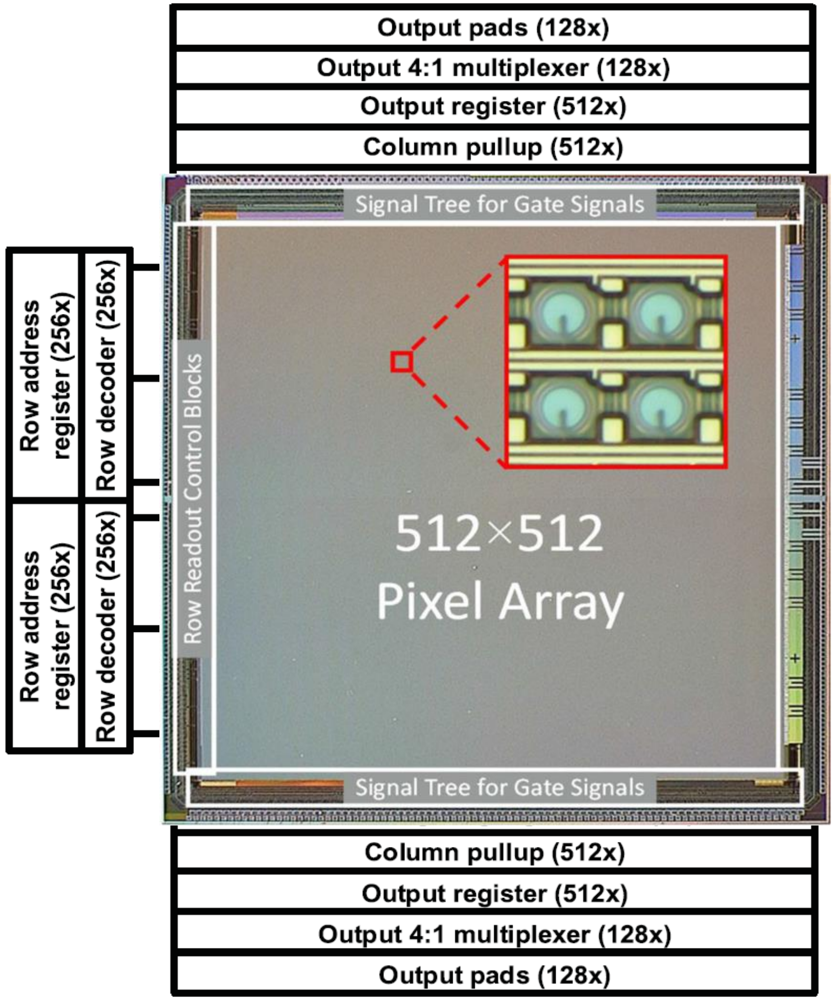
Exercise 1: Group explanation

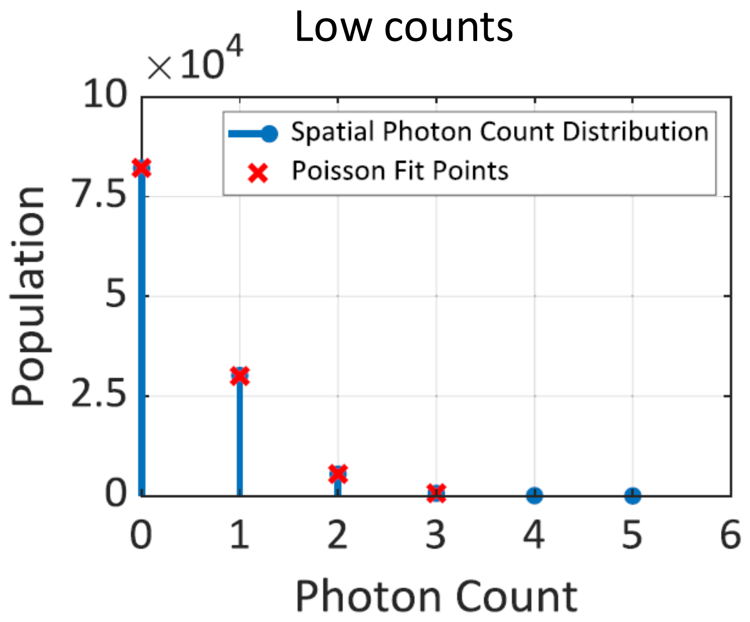
1. Divide yourself in small group (3 ppl)
2. Discuss the following example taken from the lecture, focusing on understanding what is happening.

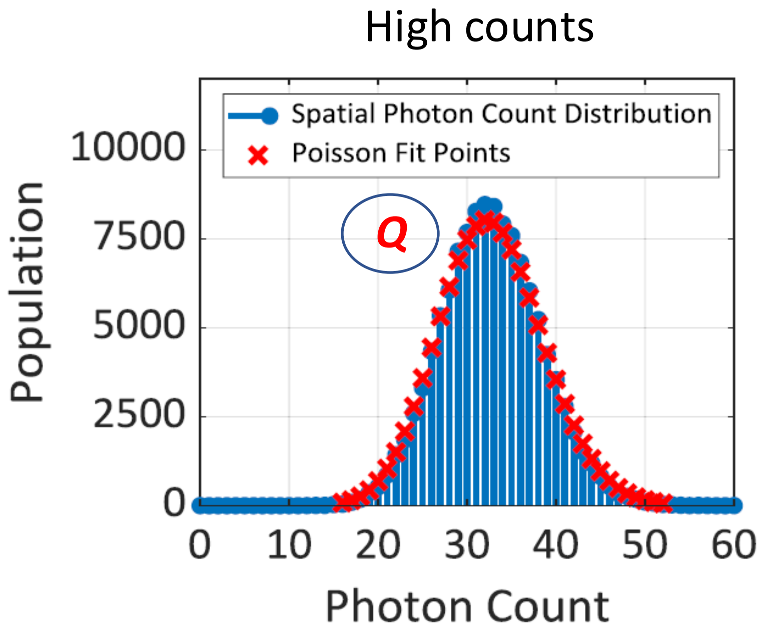
8.2.9 Example 1: Photon-flux dependent distributions

SwissSPAD2 binary SPAD imager

(intensity)

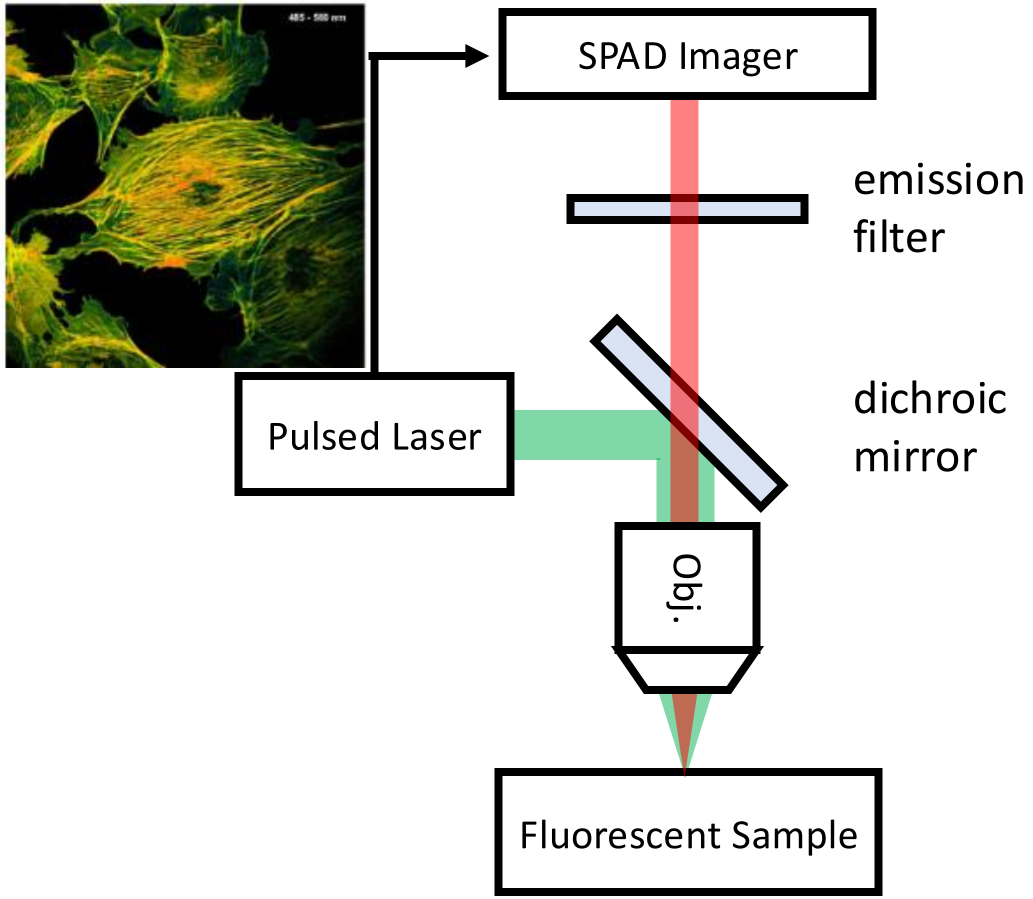


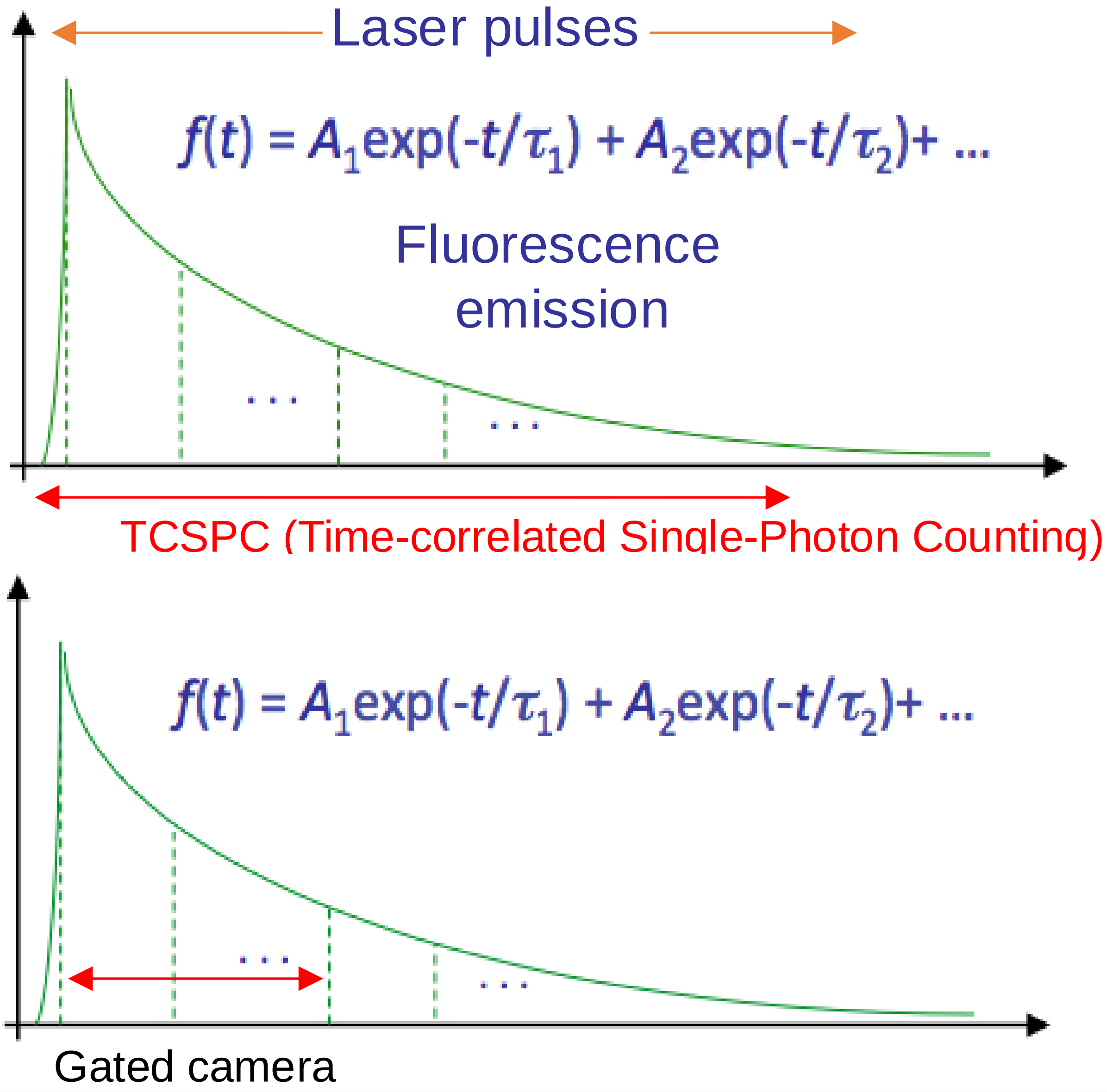




A. Ulku *et al.*, *A 512x512 SPAD Image Sensor with Integrated Gating for Widefield FLIM.* IEEE JSTQE (2019).

8.2.9 Example 2: Fluorescence Lifetime – Time-Resolved





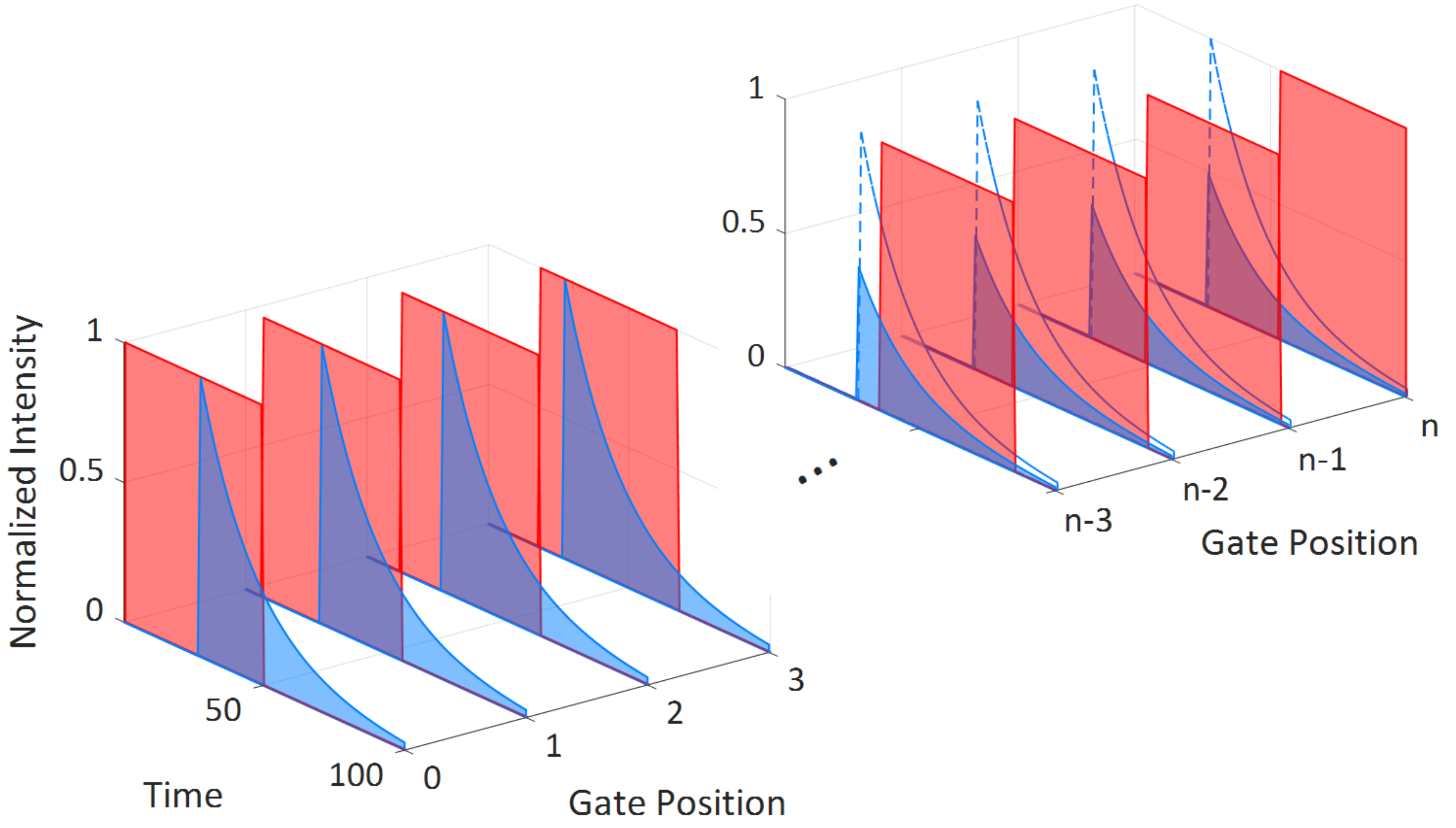
Lifetime images: the pixel time-tags all photons and calculates t_1 , t_2 , A_1

D. Li, *Strathclyde Univ.* (2016).

8.2.9 Example 2: Fluorescence Lifetime – Time-Resolved

SwissSPAD2
binary SPAD
imager

(overlapping gates)



A. Ulku *et al.*, *Large-Format Time-Gated SPAD Cameras for Real-Time Phasor-Based FLIM. EPFL Thèse 8311* (2021).

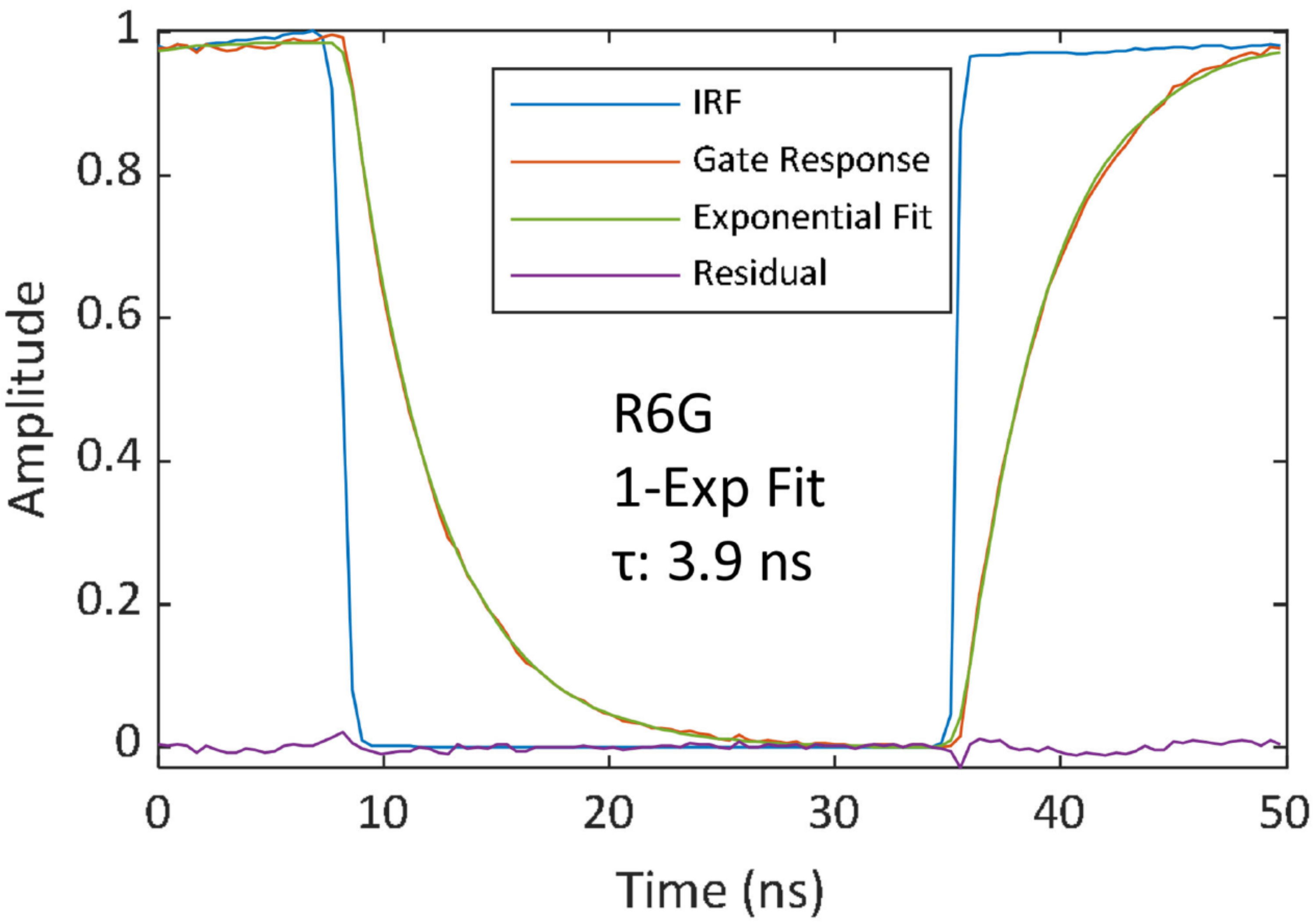
8.2.9 Example 2: Fluorescence Lifetime – Time-Resolved

SwissSPAD2
binary SPAD
imager

(overlapping gates
-> convolution)

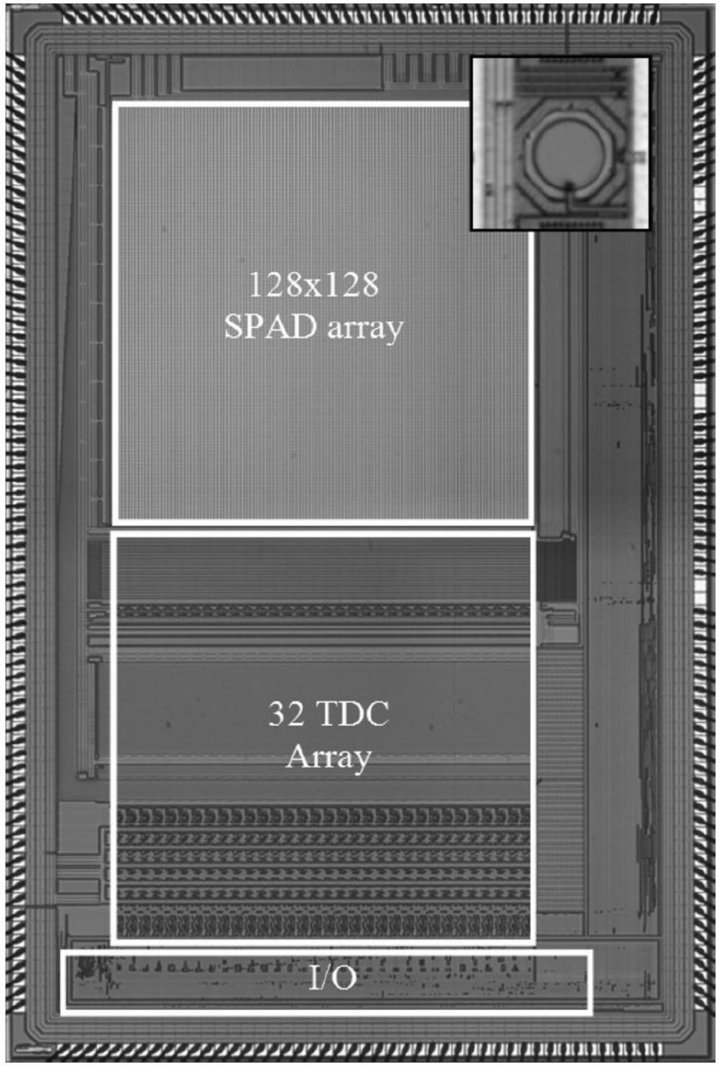
$f(t) = g(t) * IRF(t)$

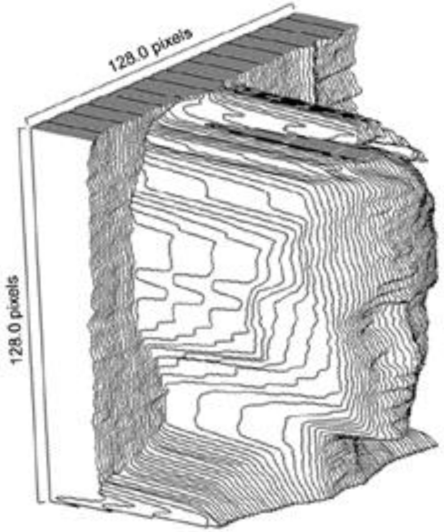
IRF: Instrument
Response Function

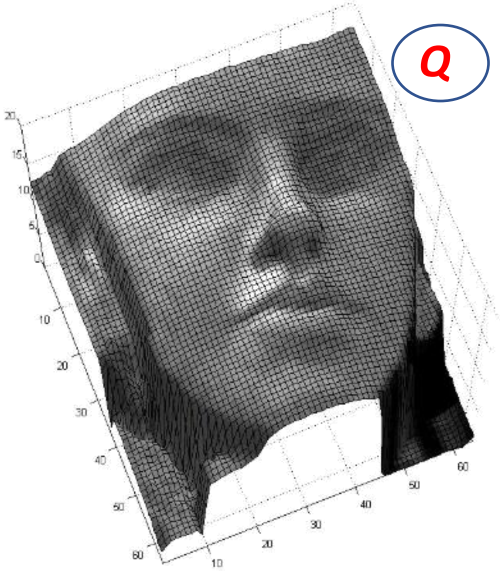


A. Ulku *et al.*, *A 512x512 SPAD Image Sensor with Integrated Gating for Widefield FLIM.* IEEE JSTQE (2019).

8.2.9 Example 3: Real Life Truths – LIDAR & Timing Jitter in SPADs







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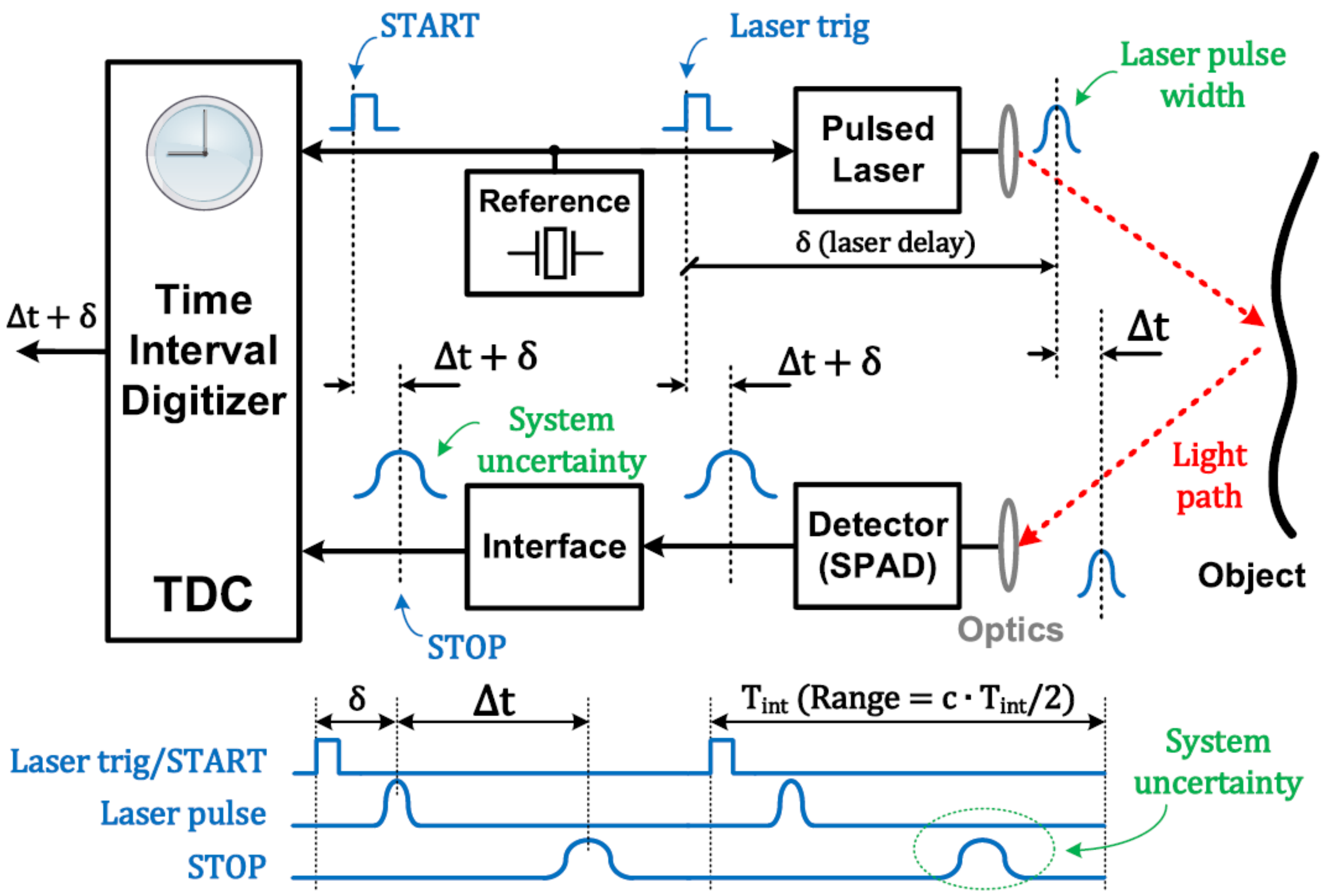
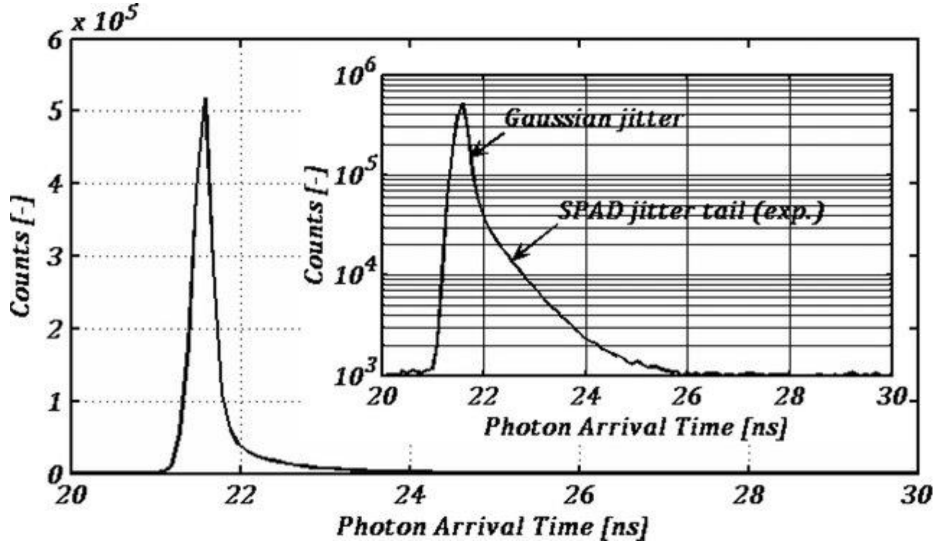
C. Niclass *et al.*, *A 128×128 Single-Photon Image Sensor With Column-Level 10-Bit Time-to-Digital Converter Array.* IEEE JSSC 43 (2008).

8.2.9 Example 3: Real Life Truths – LIDAR & Timing Jitter in SPADs



Direct SPAD illumination -> SPAD IRF (jitter noise) ->

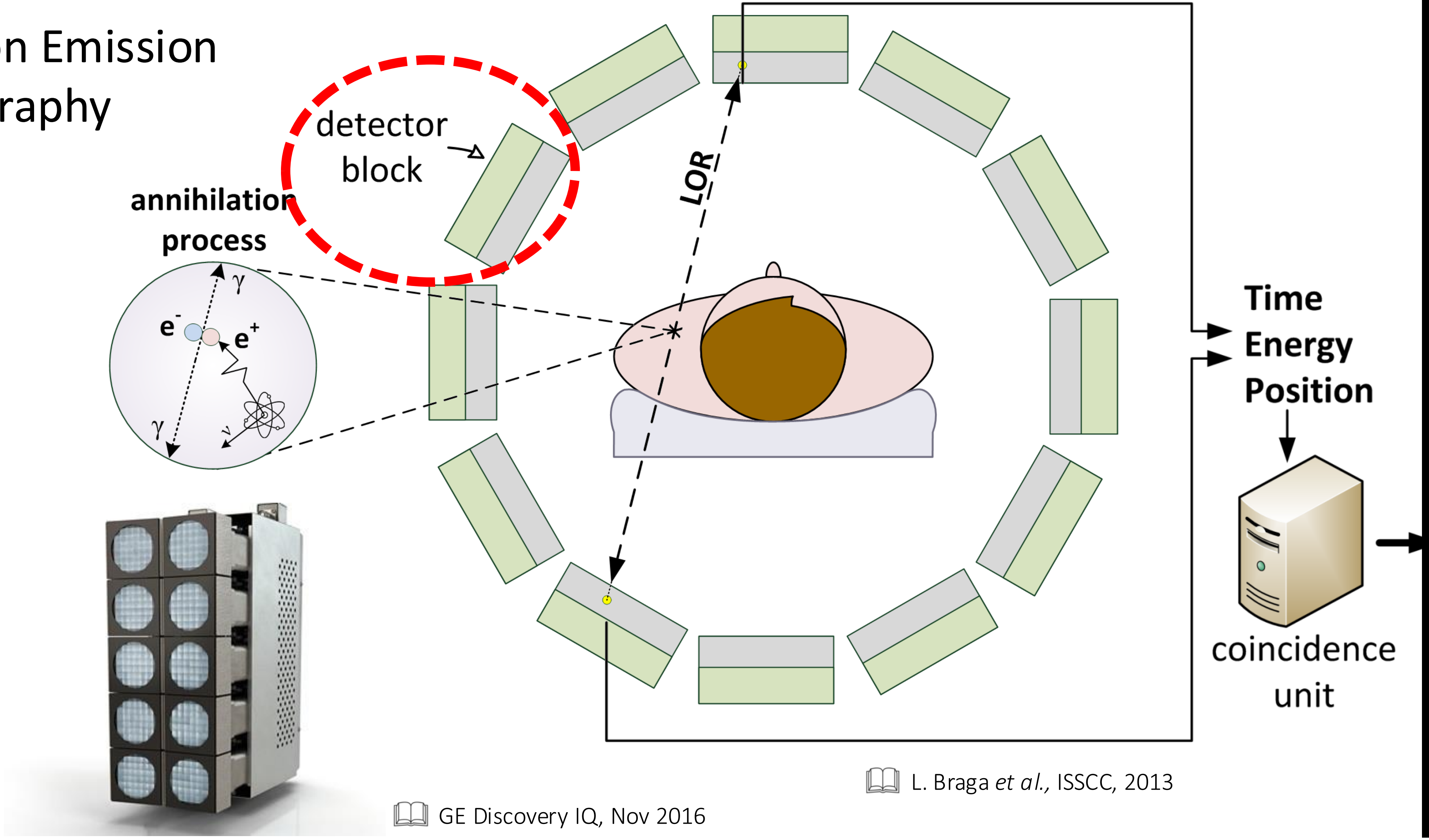
Non-Gaussian behavior of the SPADs timing uncertainty



A. R. Ximenes *et al.*, *A Modular, Direct Time-of-Flight Depth Sensor in 45/65-nm 3-D-Stacked CMOS Technology*. IEEE JSSC 54 (2019).
C. Niclass *et al.*, *A 128×128 Single-Photon Image Sensor With Column-Level 10-Bit Time-to-Digital Converter Array.* IEEE JSSC 43 (2008).

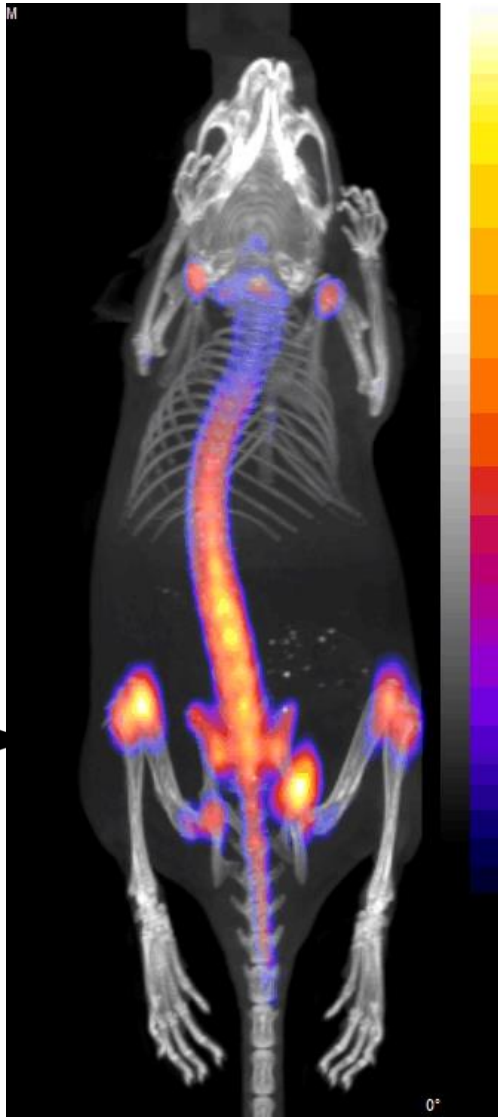
8.2.9 Example 4: Real Life Truths – Scintillation Light

Positron Emission Tomography Basics



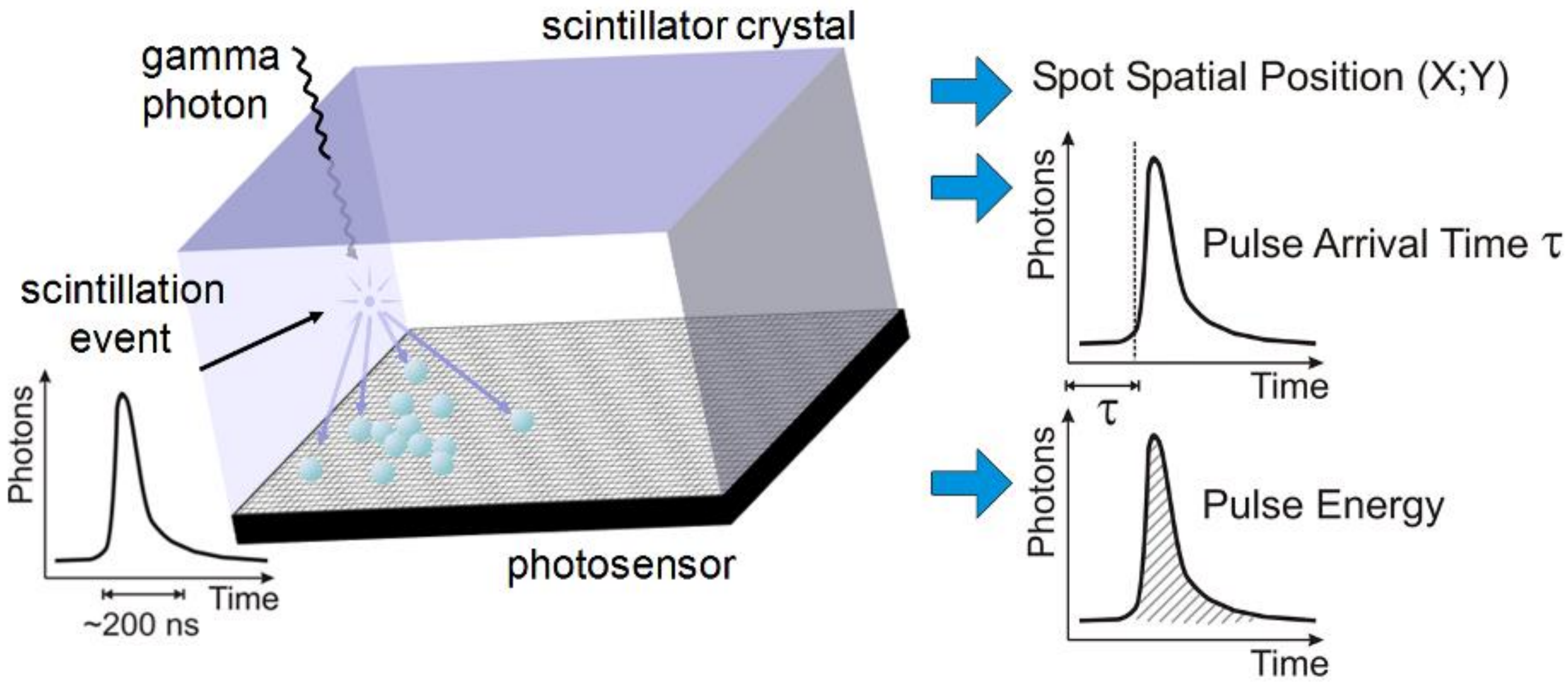
GE Discovery IQ, Nov 2016

L. Braga *et al.*, ISSCC, 2013



G. Nemeth, Mediso, Delft WS 2010

8.2.9 Example 4: Real Life Truths – Scintillation Light



R. Walker *et al.*, IISW, 2013

8.2.9 Example 4: Real Life Truths – Scintillation Light

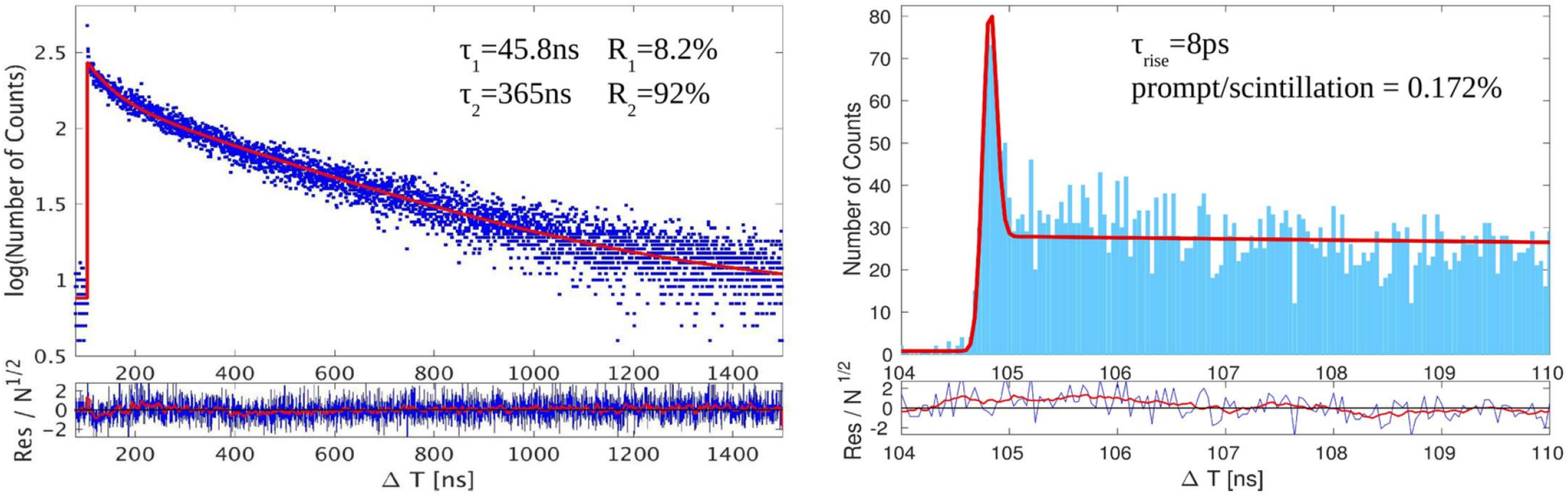


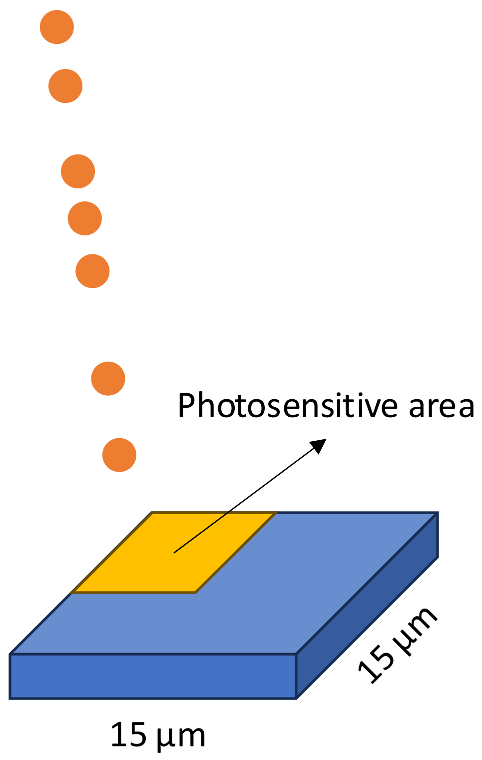
Figure 10. Scintillation decay and rise time of BGO measured with a time correlated single photon counting (TCSPC) setup using 511 keV annihilation gammas (Gundacker *et al* 2016b). The figure on the right hand side shows a pronounced Cherenkov peak at the onset of the scintillation emission with a relative abundance of 0.172% compared to the total amount of photons detected by the stop detector of the TCSPC setup.

Fast vs. "slow" scintillation photons in a heavy scintillating crystal

Gundacker S, Auffray E, Pauwels K and Lecoq P *Measurement of intrinsic rise times for various L(Y)SO and LuAG scintillators with a general study of prompt photons to achieve 10 ps in TOF-PET.* IOP Phys. Med. Biol. 61 2802–37

Exercise 2: Missed Photon Count

- A single photon detector (active area of 15x15 μm^2, fill factor of 20%) is illuminated with a continuous wave red laser (633 nm) with a uniform surface power density of 2 μW/cm^2.
- The detector has an average photon detection probability (PDP) at 633 nm of 35%.
- If the dead time of the detector (t_D, time for the detector to recover operation after clicking) is of 2 ns, what is the probability that photons are missed during detector's dead time?



Single Photon Energy:

$$E_{ph} = h\nu = h\frac{c}{\lambda}$$

Exercise 3: Moment Generating Function

- Obtain the moment generating function of the normal distribution $X \sim \mathcal{N}(\mu, \sigma^2)$. Calculate the first three moments.

- The moment generating function (MGF) of a RV X is defined as:

$$\text{MGF: } \phi(t) = E\{e^{tX}\} = \begin{cases} \sum_x e^{tx} p_X(x), & \text{if } X \text{ is discrete } * \\ \int_{-\infty}^{\infty} e^{tx} f_X(x)\, dx\,, & \text{if } X \text{ is continuous } * \end{cases}$$

Homework 1: Rare disease

- A rare disease affects 1 person every 100'000. The SV researchers in EPFL are developing a new test method, which shows a sensitivity of 0.8 and a specificity of 0.9 in the 3rd phase trial. What is the probability that a patient is affected by this disease if the result is positive in the real world?
  - NB: the definition of sensitivity and specificity is given by the confusion matrix below

Predicted Class

		Positive	Negative	
Actual Class	**Positive**	True Positive (TP)	False Negative (FN) **Type II Error**	**Sensitivity** $\frac{TP}{(TP+FN)}$
	Negative	False Positive (FP) **Type I Error**	True Negative (TN)	**Specificity** $\frac{TN}{(TN+FP)}$
		Precision $\frac{TP}{(TP+FP)}$	**Negative Predictive Value** $\frac{TN}{(TN+FN)}$	**Accuracy** $\frac{TP+TN}{(TP+TN+FP+FN)}$

https://manisha-sirsat.blogspot.com/2019/04/confusion-matrix.html

Homework 2: Skew and Kurtosis

- Using the MGF, demonstrate that the skew of the normal distribution is zero.

- Then, calculate the kurtosis of the exponential.

Homework 3: (Matlab) distributions

- Reproduce with Matlab the different Random Variable distributions encountered in the Week 2 lecture.

Homework 4: Usefulness of Bayes' Theorem

- A company produces single-photon cameras with three production lines: the first one (line A) has 10% of defective devices, the second one (line B) 20% and the third one (line C) 30% (not a very reliable company…).
- Usually, these three production lines cover respectively 15%, 35% and 50% of the total production. We bought a device and we found it defective.
- What is it the probability that the defective device is from line C?